

APPLICABLE ANALYSIS AND DISCRETE MATHEMATICS

available online at <http://pefmath.etf.rs>APPL. ANAL. DISCRETE MATH. **11** (2017), 74–80.DOI: <https://doi.org/10.2298/AADM161111033R>MORE ON GRAPHS WITH JUST THREE DISTINCT
EIGENVALUES*Peter Rowlinson**In honour of Dragoš Cvetković on the occasion of his 75th birthday.*

Let G be a connected non-regular non-bipartite graph whose adjacency matrix has spectrum $\rho, \mu^{(k)}, \lambda^{(l)}$, where $k, l \in \mathbb{N}$ and $\rho > \mu > \lambda$. We show that if μ is non-main then $\delta(G) \geq 1 + \mu - \lambda\mu$, with equality if and only if G is of one of three types, derived from a strongly regular graph, a symmetric design or a quasi-symmetric design (with appropriate parameters in each case).

1. INTRODUCTION

Let G be a graph of order n with $(0, 1)$ -adjacency matrix A . An eigenvalue σ of A is said to be an eigenvalue of G , and σ is a *main* eigenvalue if the eigenspace $\mathcal{E}_A(\sigma)$ is not orthogonal to the all-1 vector in \mathbb{R}^n . Always the largest eigenvalue, or *index*, of G is a main eigenvalue, and it is the only main eigenvalue if and only if G is regular. We say that G is an *integral* graph if every eigenvalue of G is an integer; and G is a *biregular* graph if it has just two different degrees. We use the notation of the monograph [6], where the basic properties of graph spectra can be found in Chapter 1.

Let \mathcal{C}_1 be the class of connected graphs with just three distinct eigenvalues, and let \mathcal{C}_2 be the class of connected graphs with exactly two main eigenvalues. It is an open problem to determine all the graphs in \mathcal{C}_1 , and another open problem to determine all the graphs in \mathcal{C}_2 . Here we continue the investigation of graphs in

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$\mathcal{C}_1 \cap \mathcal{C}_2$ begun in [14]. Independently the authors of [3] investigated the biregular graphs in \mathcal{C}_1 , and it is not difficult to see that these are precisely the graphs in $\mathcal{C}_1 \cap \mathcal{C}_2$: this follows from [3, Theorem 4.3(i)] and [14, Lemma 2.2]. Some examples of biregular graphs in $\mathcal{C}_2 \setminus \mathcal{C}_1$ (of order 16) can be found in [10, Table 1].

In [14] it was noted that any graph G in $\mathcal{C}_1 \cap \mathcal{C}_2$ is either integral or complete bipartite. Examples include the following, where G has spectrum $\rho, \mu^{(k)}, \lambda^{(l)}$ with $\rho > \mu > \lambda$.

- (1) G is of *conical type*: here G is the cone over a strongly regular graph with parameters $\lambda^2\mu + \lambda^2 - \lambda\mu, \mu - \lambda\mu, 2\mu + \lambda, \mu$ (see [12, 14]);
- (2) G is of *symmetric type*: here G is obtained from the incidence graph of a symmetric $2-(q^3 - q + 1, q^2, q)$ design by adding all edges between blocks (see [7]);
- (3) G is of *affine type*: here G is obtained from the incidence graph of an affine $2-(q^3, q + 1, q^2, q)$ design by adding all edges between intersecting blocks (see [8]).

There are infinitely many graphs of each type. The graphs of conical type include the cones $K_1 \nabla H$, where H is the Petersen graph ($\mu = 1, \lambda = -2$), the Gewirtz graph ($\mu = 2, \lambda = -4$) or a Chang graph ($\mu = 4, \lambda = -2$). For graphs of symmetric type we have $\rho = q^3, \mu = q - 1$ and $\lambda = -q$, and for graphs of affine type we have $\rho = q^3 + q^2 + q, \mu = q$ and $\lambda = -q$. If G is a graph of any of these three types then μ is the non-main eigenvalue, and the minimum degree $\delta(G)$ is equal to $1 + \mu - \lambda\mu$.

To formulate a partial converse implicit in [14, Section 4] we extend the construction in (3) to any quasi-symmetric design which has 0 as one of its two intersection numbers. If \mathcal{D} is such a design then we denote by $G_{\mathcal{D}}$ the graph obtained from the incidence graph of \mathcal{D} by adding all edges between intersecting blocks: thus $G_{\mathcal{D}}$ is the union of the block graph of \mathcal{D} with the incidence graph of \mathcal{D} . We refer to the graphs $G_{\mathcal{D}}$ as graphs of *quasi-symmetric type*. It follows from [14, Theorem 4.2] that if μ is non-main and $\delta(G) = 1 + \mu - \lambda\mu$ then G is of conical, symmetric or quasi-symmetric type; moreover if G is of quasi-symmetric type, with $G = G_{\mathcal{D}}$, then the parameters of \mathcal{D} are specified rational functions of λ and μ , and the intersection numbers are 0 and $-\lambda$. (We have strong integrality conditions as a consequence.) In Section 3 we show conversely that for any quasi-symmetric design \mathcal{D} with these parameters and intersection numbers, the graph $G_{\mathcal{D}}$ lies in $\mathcal{C}_1 \cap \mathcal{C}_2$, with μ non-main and $\delta(G_{\mathcal{D}}) = 1 + \mu - \lambda\mu$. We shall see also that $\delta(G) \geq 1 + \mu - \lambda\mu$ whenever $G \in \mathcal{C}_1 \cap \mathcal{C}_2$ and μ is non-main. (For an example with strict inequality here we may take G to be the unique maximal exceptional graph of order 36 [5, Section 6.1]: for this graph we have $\delta(G) = 18, \mu = 5$ and $\lambda = -2$.)

To state one further observation, recall from [1] that a *multiplicative* graph is a connected graph whose distinct eigenvalues are $\rho, \mu, -\mu$, where $\rho > \mu > 0$. For such graphs, $\lambda = -\mu$, and in Section 4 we show that if $\delta(G) = 1 + \mu + \mu^2$ then $-\mu$ is a main eigenvalue. We deduce that the only multiplicative graphs in $\mathcal{C}_1 \cap \mathcal{C}_2$ with $\delta(G) = 1 + \mu + \mu^2$ are (a) the graphs of affine type, and (b) the cones over a strongly regular graph with parameters $\mu^3 + 2\mu^2, \mu^2 + \mu, \mu, \mu$. It was already shown in [1] that the graphs of type (b) lie in \mathcal{C}_1 .

2. PRELIMINARIES

Here we note three results required in subsequent sections.

Lemma 2.1 [14, Lemma 2.2]. *A graph G in $\mathcal{C}_1 \cap \mathcal{C}_2$ has exactly two distinct degrees (say d_1, d_2), and these degrees determine an equitable bipartition of G . Moreover, if G has spectrum $\rho, \mu^{(k)}, \lambda^{(l)}$, where $\rho > \mu > \lambda$, then $d_h = \alpha_h^2 - \lambda\mu$, where $\alpha_h > 0$ ($h = 1, 2$) and either*

- (a) μ is non-main and $\alpha_1\alpha_2 = -\lambda(\mu + 1)$, or
- (b) λ is non-main and $\alpha_1\alpha_2 = -\mu(\lambda + 1)$.

Next recall that a quasi-symmetric design is a 2-design \mathcal{D} for which the number of points in the intersection of two blocks takes exactly two values. We say that \mathcal{D} is a quasi-symmetric (a, b, c, d, e) -design with intersection numbers α and β ($\alpha < \beta$) if it has a points, b blocks, each point lies in c blocks, each block contains d points, any two points lie in exactly e blocks, and two blocks intersect in α or β points. The *block graph* of \mathcal{D} has the blocks of \mathcal{D} as its vertices, with blocks adjacent if and only if they intersect in β points.

Theorem 2.2 [11, Theorem 8.3.14]. *The block graph of a quasi-symmetric (a, b, c, d, e) -design with intersection numbers α and β ($\alpha < \beta$) is strongly regular with spectrum $\xi, \eta^{(a-1)}, \zeta^{(b-a)}$, where*

$$\xi = \frac{(c-1)d - \alpha(b-1)}{\beta - \alpha}, \quad \eta = \frac{c - d - e + \alpha}{\beta - \alpha}, \quad \zeta = \frac{\alpha - d}{\beta - \alpha}.$$

To formulate [14, Theorem 4.2(c)] in terms of a quasi-symmetric design, we define

$$(1) \quad f_1(\lambda, \mu) = \frac{\lambda(\lambda + \lambda\mu - \lambda^2\mu + \mu)}{\mu + 1}, \quad f_2(\lambda, \mu) = \frac{(1 + \mu - \lambda\mu)(\lambda + \lambda\mu - \lambda^2\mu + \mu)}{\lambda(\mu + 1)}.$$

Now we have

Theorem 2.3 [14, Theorem 4.2]. *Let G be a connected non-regular non-bipartite graph whose distinct eigenvalues are ρ, μ, λ where $\rho > \mu > \lambda$. If μ is non-main and $\delta(G) = 1 + \mu - \lambda\mu$ then G is of conical, symmetric or quasi-symmetric type. Moreover, if G is of quasi-symmetric type then $G = G_{\mathcal{D}}$ where \mathcal{D} is a quasi-symmetric $(f_1(\lambda, \mu), f_2(\lambda, \mu), 1 + \mu - \lambda\mu, \lambda^2, 1 + \mu)$ -design with intersection numbers 0 and $-\lambda$.*

3. GRAPHS OF QUASI-SYMMETRIC TYPE

Our main objective in this section is to prove the converse of Theorem 2.3. It has already been noted that the hypotheses are satisfied if G is of conical or symmetric type. Accordingly suppose that $G = G_{\mathcal{D}}$, where \mathcal{D} is a quasi-symmetric

$(f_1(\lambda, \mu), f_2(\lambda, \mu), 1 + \mu - \lambda\mu, \lambda^2, 1 + \mu)$ -design with intersection numbers 0 and $-\lambda$. Note that μ is a positive integer and λ is a negative integer. We show first that the distinct eigenvalues of G are ρ, μ, λ , where $\rho = -\lambda(1 + \mu - \lambda\mu)$.

Let V_1 be the set of blocks of \mathcal{D} , let V_2 be the set of points of \mathcal{D} , and let G_1 be the block graph of \mathcal{D} , that is, the graph induced by V_1 . From Theorem 2.2 we know that G_1 is strongly regular with eigenvalues $\nu, \lambda + \mu, \lambda$, where $\nu = \lambda\mu(\lambda - 1)$. Now let $m_1(x) = (x - \nu)(x - \lambda - \mu)(x - \lambda)$, $m(x) = (x - \rho)(x - \mu)(x - \lambda)$. Let G have adjacency matrix $A = \begin{pmatrix} A_1 & B^\top \\ B & O \end{pmatrix}$, and let $m(A) = \begin{pmatrix} X & Y^\top \\ Y & Z \end{pmatrix}$, both partitioned in accordance with $V_1 \cup V_2$.

From the parameters and intersection numbers of \mathcal{D} we see that $B^\top B = \lambda^2 I - \lambda A_1$ and $BB^\top = -\lambda\mu I + (\mu + 1)J$, where each entry of J is 1. On expanding $m(A)$ and substituting for ρ , $B^\top B$ and BB^\top , we find that $X = m_1(A_1)$, and so $X = O$. Again, $Y^\top = (A_1^2 - (\lambda + \mu + \nu)A_1 + (\lambda + \mu)\nu I)B^\top$, from which it follows that $Y^\top B = -\lambda m_1(A_1) = O$. Since BB^\top is invertible, we have $Y^\top = O$.

Further calculation shows that $Z = BA_1B^\top - \mu(1 - \lambda + \lambda^2)(1 + \mu)J + \lambda\mu(\lambda + \mu)I$. In what follows, we write \mathbf{j} for an all-1 vector, its length determined by context. We have $BA_1B^\top \mathbf{j} = \lambda^2 BA_1 \mathbf{j} = \lambda^2 \nu B \mathbf{j} = \lambda^3 \mu(\lambda - 1)(1 + \mu - \lambda\mu)\mathbf{j}$, and it follows that

$$Z\mathbf{j} = \lambda^3 \mu(\lambda - 1)(1 + \mu - \lambda\mu)\mathbf{j} - \mu(1 - \lambda + \lambda^2)(1 + \mu)f_1(\lambda, \mu)\mathbf{j} + \lambda\mu(\lambda + \mu)\mathbf{j} = \mathbf{0}.$$

Now BB^\top commutes with A_1 and hence with BA_1B^\top . The matrices BB^\top and BA_1B^\top are therefore simultaneously diagonalizable. Since \mathbf{j} is an eigenvector of both of these matrices, there exist $f_1(\lambda, \mu)$ pairwise orthogonal eigenvectors, including \mathbf{j} , which are eigenvectors of both BB^\top and BA_1B^\top . Let \mathbf{x} be such a vector orthogonal to \mathbf{j} . We have $Z\mathbf{x} = BA_1B^\top \mathbf{x} + \lambda\mu(\lambda + \mu)\mathbf{x}$ and $BB^\top \mathbf{x} = -\lambda\mu\mathbf{x}$. Now $\lambda\mu B^\top \mathbf{x} = (B^\top B)B^\top \mathbf{x} = (\lambda^2 - \lambda)A_1B^\top \mathbf{x}$, whence $A_1B^\top \mathbf{x} = (\lambda + \mu)B^\top \mathbf{x}$ and $BA_1B^\top \mathbf{x} = -\lambda\mu(\lambda + \mu)\mathbf{x}$. It follows that Z annihilates all $f_1(\lambda, \mu)$ pairwise orthogonal eigenvectors, and hence that $Z = O$. We deduce that $m(A) = 0$ and hence that the distinct eigenvalues of G are ρ, λ, μ as required. Now we can prove:

Theorem 3.1. *Let G be a connected non-regular non-bipartite graph whose distinct eigenvalues are ρ, μ, λ where $\rho > \mu > \lambda$. If μ is non-main then $\delta(G) \geq 1 + \mu - \lambda\mu$ with equality if and only if G is of conical, symmetric or quasi-symmetric type $G_{\mathcal{D}}$, where \mathcal{D} is a quasi-symmetric $(f_1(\lambda, \mu), f_2(\lambda, \mu), 1 + \mu - \lambda\mu, \lambda^2, 1 + \mu)$ -design with intersection numbers 0 and $-\lambda$. (The functions f_1, f_2 are defined in Eq.(1).)*

Proof. As noted in [7, 14], the adjacency matrix A of G satisfies the equation

$$(2) \quad (A - \mu I)(A - \lambda I) = \mathbf{a}\mathbf{a}^\top,$$

where $A\mathbf{a} = \rho\mathbf{a}$ and each entry of \mathbf{a} is positive. By Lemma 2.1, G has two degrees, say d_1 and d_2 . Moreover $d_h = \alpha_h^2 - \lambda\mu$ ($h = 1, 2$) where, with a suitable ordering of vertices, $\mathbf{a} = \begin{pmatrix} \alpha_1 \mathbf{j} \\ \alpha_2 \mathbf{j} \end{pmatrix}$, $\alpha_1 > \alpha_2$ and $\alpha_1 \alpha_2 = -\lambda(\mu + 1)$. Let i, j be adjacent vertices with $\deg(i) = d_1, \deg(j) = d_2$. From Eq.(2), the number of i - j walks of length 2 is

$\alpha_1\alpha_2 + \lambda + \mu$, that is, $-\lambda\mu + \mu$. Now $-\lambda\mu + \mu \leq \deg(j) - 1 = \delta(G) - 1$, and so $\delta(G) \geq 1 + \mu - \lambda\mu$.

In the light of Theorem 2.3 and the argument above, it remains to show that when $G = G_{\mathcal{D}}$, the eigenvalue μ is non-main and $\delta(G) = 1 + \mu - \lambda\mu$. In this case let $\begin{pmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{pmatrix}$ be the divisor matrix D determined by the equitable partition $V_1 \dot{\cup} V_2$, where V_1 is the set of blocks of \mathcal{D} and V_2 is the set of points of \mathcal{D} . We have $r_{11} = \nu = \lambda\mu(\lambda - 1)$ and $r_{22} = 0$, and so D has trace $\rho + \lambda$. It follows from [14, Theorem 3.9.9] that G has exactly two main eigenvalues; moreover, since ρ is one main eigenvalue of G , the other is λ .

Finally, the vertices in V_1 have degree $\nu + \lambda^2 = -\lambda(-\lambda - \lambda\mu + \mu)$, while those in V_2 have the smaller degree $1 + \mu - \lambda\mu$. Thus $\delta(G) = 1 + \mu - \lambda\mu$ and the proof of the theorem is complete. \square

4. FURTHER REMARKS

(1) A question which remains is whether there exist any graphs $G_{\mathcal{D}}$ of quasi-symmetric type that are not of affine type. As noted in [14], any candidate has $\lambda + \mu > 0$; moreover λ and μ are not coprime. The candidate with smallest $\mu - \lambda$ has order 625, with $\mu = 9$ and $\lambda = -6$. By Theorem 3.1 such a graph exists if and only if there exists a quasi-symmetric $(225, 400, 64, 36, 10)$ -design \mathcal{D} . Then G_1 , the block graph of \mathcal{D} , is strongly regular with parameters 400, 378, 357, 360; according to [2], the existence of such a strongly regular graph is unknown. We note that the parameters of G_1 and \mathcal{D} satisfy the four necessary conditions for the existence of \mathcal{D} formulated in [13, Theorem 6].

(2) The authors of [9] investigate the strongly regular graphs G with a *strongly regular decomposition*. This means that $V(G) = V_1 \dot{\cup} V_2$, where (for $i = 1, 2$) the graph G_i induced by V_i is a strongly regular graph or a clique or a co-clique: then $V_1 \dot{\cup} V_2$ is an equitable partition. In the case that one of G_1, G_2 is a co-clique, the decomposition is said to be *improper*.

One can seek to generalize the results of [9] by requiring instead that G is a graph in \mathcal{C}_1 with an equitable bipartition and an associated strongly regular decomposition. If G is not strongly regular then $G \in \mathcal{C}_1 \cap \mathcal{C}_2$, and the graphs of Theorem 3.1 are examples of such a graph with an improper strongly regular decomposition.

Now suppose that G is a graph in $G \in \mathcal{C}_1 \cap \mathcal{C}_2$ having an improper strongly regular decomposition, with G_2 a non-trivial co-clique and G_1 a strongly regular graph with parameters n_1, r_1, e_1, f_1 . We make two comments using the notation of Section 3. First, μ is the non-main eigenvalue of G for otherwise $-r_{12}r_{21} = \det(D) = \rho\mu > 0$, a contradiction. Secondly, if $i, j \in V_1$ ($i \neq j$) then by Eq.(2) the (i, j) -entry of A^2 is given by

$$a_{ij}^{(2)} = \begin{cases} \alpha_1^2 & \text{if } i \not\sim j, \\ \alpha_1^2 + \lambda + \mu & \text{if } i \sim j. \end{cases}$$

Similarly, if $i, j \in V_2$ ($i \neq j$) we have $a_{ij}^{(2)} = \alpha_2^2$. It follows that the neighbourhoods $\{h \in V_2 : h \sim i\}$ ($i \in V_1$) form either a symmetric 2-design or a quasi-symmetric (a, b, c, d, e) -design with intersection numbers α, β , where $a = n_2$, $b = n_1$, $c = r_{21}$, $d = r_{12}$, $e = \alpha_2^2$, $\alpha = \alpha_1^2 - f_1$ and $\beta = \alpha_1^2 + \lambda + \mu - e_1$ (cf. [9, Theorem 2.9]).

(3) Our final remark concerns multiplicative graphs, for which $\lambda = -\mu$. Let G be a connected non-regular graph with spectrum $\rho, \mu^{(k)}, -\mu^{(l)}$, where $\rho > \mu > -\mu$ and one eigenvalue is non-main. Suppose that $\delta(G) = 1 + \mu + \mu^2$.

If $-\mu$ is non-main, then we obtain a contradiction as follows. In the notation of Section 3, we have $\alpha_2^2 = 1 + \mu$ as before, and $\alpha_1\alpha_2 = \mu(\mu - 1)$ by Lemma 2.1. Then $\alpha_1^2 = \mu^2(\mu - 1)^2/(\mu + 1)$. Since $\alpha_1 \neq 0$, we deduce that $\mu = 3$. Then $\alpha_1 = 3$, $\alpha_2 = 2$, $d_1 = 18$, $d_2 = 13$. Now $d_2 < \rho < d_1$, while $\rho + k\mu + l(-\mu) = 0$. Hence 3 divides ρ , and so $\rho = 15$.

From $A\mathbf{a} = \rho\mathbf{a}$ we have $\rho\alpha_1 = r_{11}\alpha_1 + r_{12}\alpha_2$ and $\rho\alpha_2 = r_{21}\alpha_1 + r_{22}\alpha_2$. Since also $r_{11} + r_{12} = d_1$ and $r_{21} + r_{22} = d_2$ we deduce that

$$r_{12} = \frac{\alpha_1(d_1 - \rho)}{\alpha_1 - \alpha_2} = 9, \quad r_{21} = \frac{\alpha_2(d_2 - \rho)}{\alpha_2 - \alpha_1} = 4.$$

Hence $9n_1 = 4n_2$, where $n_h = |V_h|$ ($h = 1, 2$). Also $\|\mathbf{a}\|^2 = \rho^2 - \mu^2$, whence $9n_1 + 4n_2 = 216$. We deduce that $n_1 = 12$, $n_2 = 27$, $n = 39$. From the equations $15 + 3k - 3l = 0$, $1 + k + l = 39$, we obtain the contradiction $6 + 2k = 39$. This non-existence result can also be extracted from the results in [3].

We conclude that $-\mu$ is a main eigenvalue, and so μ is the non-main eigenvalue when $G \in \mathcal{C}_1 \cap \mathcal{C}_2$. We can then apply Theorem 3.1 with $\lambda = -\mu$. Then G is of conical or quasi-symmetric type. In the latter case, $f_1(-\mu, \mu) = \mu^3$ and $f_2(-\mu, \mu) = \mu^3 + \mu^2 + \mu$; moreover the block graph G_1 is strongly regular with parameters $\mu^3 + \mu^2 + \mu, \mu^3 + \mu^2, \mu^3 + \mu^2 - \mu, \mu^3 + \mu^2$. Thus $G_1 = (1 + \mu + \mu^2)K_\mu$. It follows that G is of affine type because the blocks are partitioned into $1 + \mu + \mu^2$ parallel classes, with blocks taken as parallel when they are disjoint (see [4, Theorem 1.44]). Accordingly we have:

Proposition 4.1. *let G be a connected graph whose distinct eigenvalues are $\rho, \mu, -\mu$, where $\rho > \mu > -\mu$ and just one eigenvalue is non-main. Then $\delta(G) = 1 + \mu + \mu^2$ if and only if G is either (a) a graph of affine type, or (b) the cone over a strongly regular graph with parameters $\mu^3 + 2\mu^2, \mu^2 + \mu, \mu, \mu$.*

REFERENCES

1. W. G. BRIDGES, R. A. MENA. *Multiplicative cones – a family of three eigenvalue graphs*, Aequationes Mathematicae **22** (1981), 208–214.
2. A. E. BROUWER, *A table of parameters of strongly regular graphs*, <http://www.win.tue.nl/~aeb/>.
3. X.-M CHENG, A. L. GAVRILYUK, G. R. W. GREAVES, J. H. KOOLEN. *Biregular graphs with three eigenvalues*, European J. Math, **56** (2016), 57–80.

4. P. J. CAMERON, J. H. VAN LINT: *Designs, Graphs, Codes and their Links*, Cambridge University Press, Cambridge, 1991.
5. D. CVETKOVIĆ, P. ROWLINSON, S. K. SIMIĆ: *Spectral Generalizations of Line Graphs*, Cambridge University Press, Cambridge, 2004.
6. D. CVETKOVIĆ, P. ROWLINSON, S. K. SIMIĆ: *An Introduction to the Theory of Graph Spectra*, Cambridge University Press, Cambridge, 2010.
7. E. R. VAN DAM. *Graphs with few eigenvalues, an interplay between combinatorics and algebra*, Doctoral thesis, Tilburg University, Center for Economic Research Series No. 20, 1996.
8. E. R. VAN DAM. *Nonregular graphs with three eigenvalues*, J. Combin. Theory Ser. B **73** (1998), 101–118.
9. W. H. HAEMERS, D. G. HIGMAN. *Strongly regular graphs with a strongly regular decomposition*, Linear Algebra Appl. **114/115** (1989), 379–398.
10. S. HAYAT, J. H. KOOLEN, F. LIU, Z. QIAO. *A note on graphs with exactly two main eigenvalues*, Linear Algebra Appl. **511** (2016), 318–327.
11. Y. J. IONIN, M. S. SHRIKHANDE: *Combinatorics of Symmetric Designs*, Cambridge University Press, Cambridge, 2006.
12. M. MUZYCHUK, M. KLIN. *On graphs with three eigenvalues*, Discrete Math. **189** (1998), 191–207.
13. R. M. PAWALE, M. S. SHRIKHANDE, S. M. NYAYATE: *Non-derivable strongly regular graphs from quasi-symmetric designs*, Discrete Math. **339** (2016), 759–769.
14. P. ROWLINSON. *On graphs with just three eigenvalues*, Linear Algebra Appl. **507** (2016), 462–473.

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